NYQUIST CRITERION

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

Note

→ For stability the all the roots of CE should be $1+G(s)H(s)=0$ must lie on the left hand side of s-plane.

→ It relates the no. of zeros & poles of $1+G(s)H(s)$ that lies on R.H.S of s-plane to the open loop frequency response $G(j\omega)H(j\omega)$

→ Nyquist plot is drawn after drawing the polar plot. First we have to draw polar plot then we obtained nyq. pt by drawing its mirror image by varying ω from ∞ to −∞ & no. of encirclement of pts $-1+j0$ is observed. clockwise encirclement are taken as −ve.

→ For closed loop system to be stable, the Nyquist plot $G(s)H(s)$ must encircle the point $-1+j0$ as many times as the no. of poles of $G(s)H(s)$ that are in right half of s-plane.
Clockwise encirclement is taken as −ve.

$N = P - Z$.

N = no. of encirclements of point $-1+j0$

P = no. of poles of $G(s)H(s)$ that are on right half of s-plane

For stability of closed loop Z should be zero

$\boxed{N = P.}$ clockwise encirclement = −ve.
anticlockwise " = +ve.

Q-1 $G(s)H(s) = \frac{4s+1}{s^2(s+1)(2s+1)}$ Det^n stability

Ans:- $G(j\omega)H(j\omega) = \frac{4j\omega+1}{s^2(s+1)(2s+1)}$

$$G(j\omega)H(j\omega) = \frac{4j\omega+1}{(j\omega)^2(j\omega+1)(2j\omega+1)}$$

$$= \frac{1+j4w}{-w^2(1+jw)(1+2jw)}$$

$$|M| = \frac{\sqrt{1+16w^2}}{w^2\sqrt{1+w^2}\sqrt{1+4w^2}}$$

$$\phi = \tan^{-1}4w - \tan^{-1}(0/w^2) - \tan^{-1}\left(\frac{w}{1}\right) - \tan^{-1}\left(\frac{2w}{1}\right)$$

$$= \tan^{-1}4w - \tan^{-1}(\infty) - \tan^{-1}w - \tan^{-1}2w$$

$$= \tan^{-1}4w - 180 - \tan^{-1}w - \tan^{-1}2w$$

w	$\tan^{-1}4w$	-180	$-\tan^{-1}w$	$-\tan^{-1}2w$	ϕ
0	0	-180	0	0	-180
∞	90	-180	-90	-90	-270
0.1	21.8	-180	-5.7	-11.3	-175
1	75.9	-180	-45	-63.4	-212

$m=0$, when $w=\infty$

$m=\infty$ when $w=0$

$$G(jw)H(jw) = -\left\{ \frac{1+10w^2}{w^2[(1-2w^2)^2+9w^2]} + j\,\frac{(w-8w^3)}{w^2[(1-2w^2)^2+9w^2]} \right\}$$

$$\frac{w-8w^3}{w^2[(1-2w^2)^2+9w^2]} = 0$$

$\Rightarrow$ $w-8w^3=0 \Rightarrow w=8w^3 \Rightarrow w^2 = 1/8$

$w = 0.354$ rad/sec

It touches real axis

$$-\left\{ \frac{1+10\times 0.354^2}{(0.354)^2\left[\underbrace{\left(1-2\times(0.354)^2\right)^2}_{0.561} + \underbrace{9\times(0.354)^2}_{1.127844}\right]} \right\}$$

$$= -\left\{ \frac{2.25316}{0.2117} \right\}$$

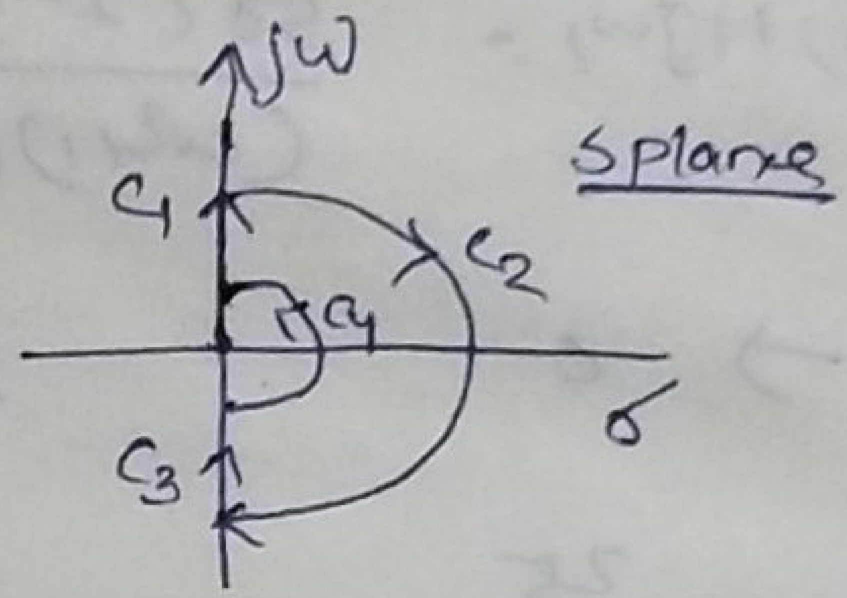
$$m = -10.64$$

Using Nyquist criterion investigate the stability of closed loop system whose $G(s)H(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$ (Page NO - 439 of Asfaq Hussain)

Ans :- $G(s)H(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$

$\therefore G(s)H(s)\Big|_{s=j\omega} = G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$

$\therefore G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$



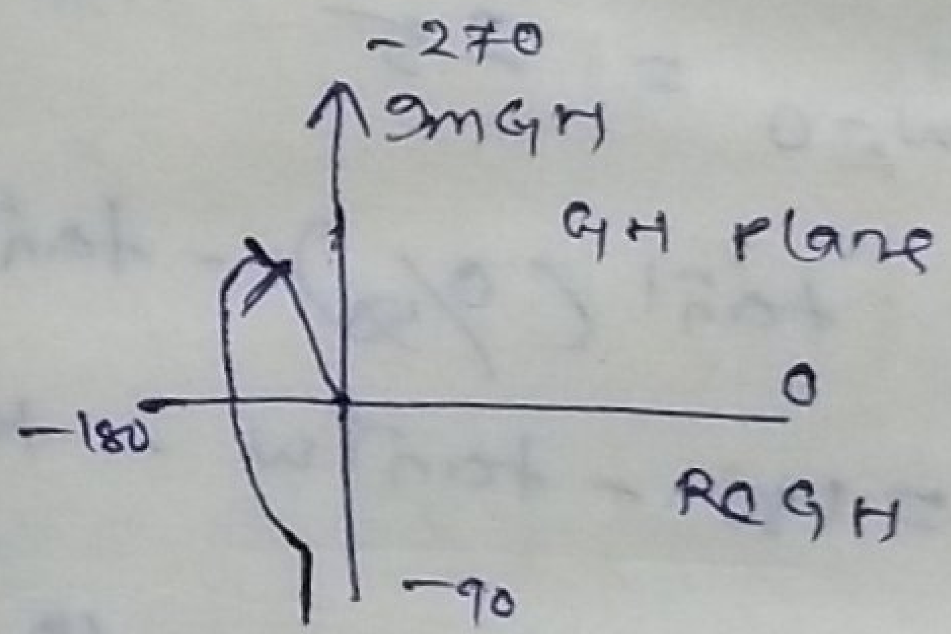
i) Mapping of section C_1 $\quad \omega \to (0 \text{ to } \infty)$

$|G(j\omega)H(j\omega)| = \frac{K}{\omega\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}}$

$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{0}{K}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$

$\Rightarrow \angle G(j\omega)H(j\omega) = 0 - \pi/2 - \tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$

$= -\pi/2 - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$



Here $\omega \to 0$ to ∞

ω	$\lvert M\rvert$	$\angle\phi$
0	∞	$-\pi/2$
∞	0	$-3\pi/2$

(ii) Mapping of section C_2

Section C_2 in s-plane is a semicircle of infinite radius

Here mapping is done putting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$

So $G(s)H(s) = \lim_{R\to\infty} \frac{K}{Re^{j\theta}(1+RT_1e^{j\theta})(1+RT_2e^{j\theta})}$

$= \lim_{R\to\infty} \frac{K}{R\cdot e^{j\theta}\cdot RT_1e^{j\theta}\cdot RT_2e^{j\theta}}$ $\quad \left(RT_1e^{j\theta}+1 \approx RT_1e^{j\theta},\ RT_2e^{j\theta}+1 \approx RT_2e^{j\theta}\right)$

$= \lim_{R\to\infty} \frac{K}{R^3 e^{3j\theta}\cdot T_1T_2}$

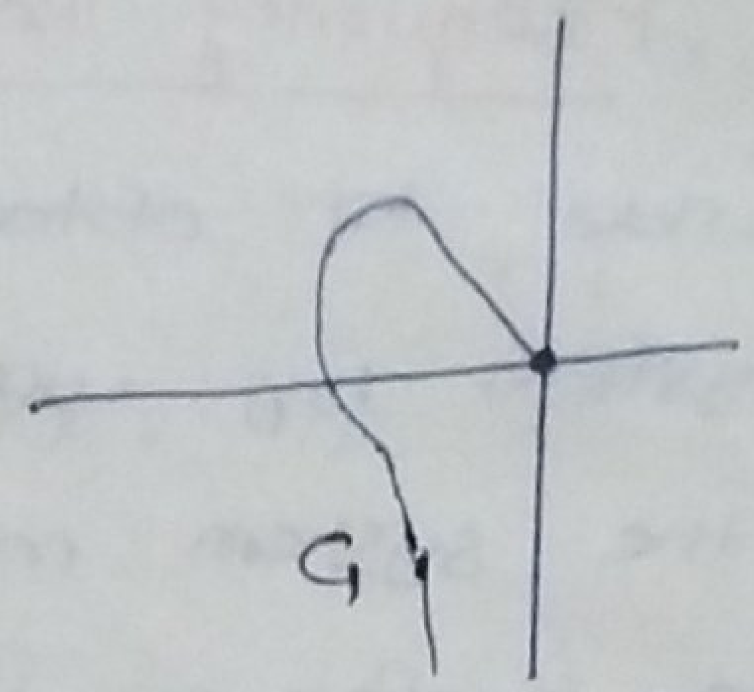
$= \lim_{R\to\infty} \frac{K}{R^3 T_1T_2\, e^{3j\theta}}$

$= \lim_{R\to\infty} \frac{K}{R^3 T_1T_2\, e^{j3\theta}} \approx 0\, e^{-j3\theta} \approx 0\angle -3\theta$

here $\theta \to +90$ to -90

so New $G(s)H(s)$ varies from

θ	$\lvert M \rvert$	$\angle\phi$
$+90$	0	$-3\pi/2$
-90	0	$+3\pi/2$



(so C_2 is mapped to origin)

(iii) Mapping of C_3 $\quad \omega \to -\infty$ to 0

$$\left|G(j\omega)H(j\omega)\right| = \frac{K}{\omega\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -\pi/2 + \pi/2 + \pi/2$$

ω	$\lvert M \rvert$	ϕ
$-\infty$	0	~~$+3\pi/2$~~ $+\pi/2$, $3\pi/2$
0	∞	$-\cancel{\pi/2}$ $\pi/2$

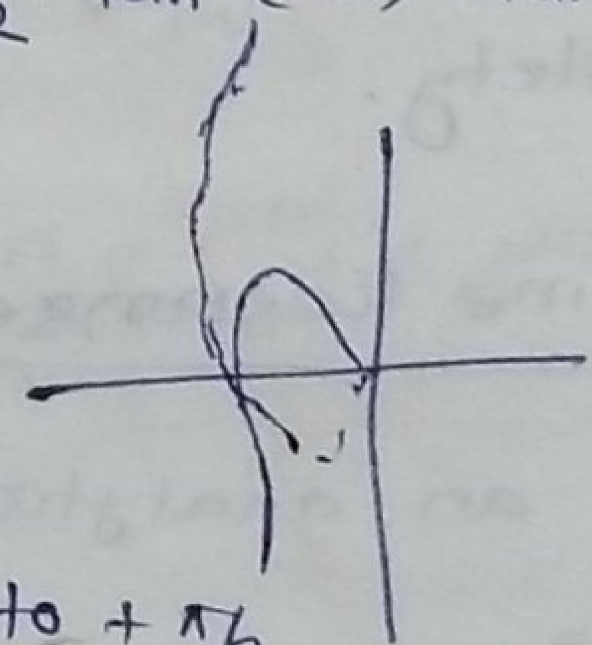
$\infty \to \left(-\pi/2 - (-\pi/2) - (-\pi/2)\right)$

$= +\pi/2 \quad \approx -3\pi/2$

$\omega \to -\infty$

$\phi = -\pi/2 - \tan^{-1}(-\infty) - \tan^{-1}(-\infty)$

It is the mirror image of C_1



(iv) Mapping of C_4

$\theta \to -\pi/2$ to $+\pi/2$

putting $s = \epsilon e^{j\theta}$

$$G(s)H(s) = \lim_{\epsilon \to 0} \frac{K}{\epsilon e^{j\theta}(1+\epsilon e^{j\theta}T_1)(1+T_2\epsilon e^{j\theta})}$$

$$= \lim_{\epsilon \to 0} \frac{K}{\epsilon e^{j\theta}(1+0)(1+0)}$$

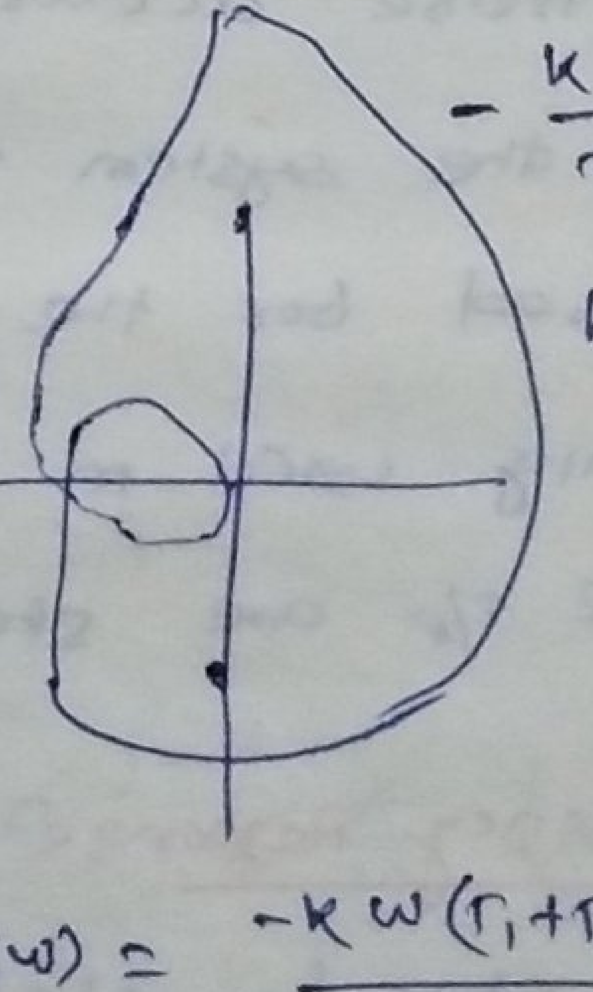
$$= \infty \cdot e^{-j\theta}$$

Stability

$Z = N + P$

$= N + 0$

θ	M	$\angle\phi$ = phase Angle
$-\pi/2$	∞	$\pi/2$
$+\pi/2$	∞	$-\pi/2$



$-\frac{KT_1T_2}{T_1+T_2} < -1$

$K < \frac{T_1+T_2}{T_1T_2}$

Intersection point at real axis

$$G(j\omega)H(j\omega) = \frac{-K\omega(T_1+T_2)}{\omega(1-\omega^2T_1^2)(1-\omega^2T_2^2)} + j\frac{K(1-\omega^2T_1T_2)}{\omega(1-\omega^2T_1^2)(1-\omega^2T_2^2)}$$

Put img $G(s)H(s) = 0$

find value of ω

put it on real part find the intersection pt

$\omega = \pm\frac{1}{\sqrt{T_1T_2}}$ $\quad \omega = \frac{1}{\sqrt{T_1T_2}}$ (+ve value) put it on real part

$\therefore$ Intersection pt $= -KT_1T_2/(T_1+T_2)$ $\quad$ ~~$G(j\omega)H(j\omega)$~~ $=$

Frequency Response Analysis

It is possible to obtain an idea about the performance of a given system by using time domain analysis. If the order of the system increases, it is very difficult to analyse the system in time domain approach. Frequency domain analysis is other analytic method that can be used for analysis & design. The comparison of time domain & frequency domain approaches can be done on the basis of relative stability of the system. The Routh Hurwitz criterion (Time domain analysis) determines the stability of a system. But for determining the relative stability it requires repeated applications of the criterion. On the other hand Nyquist criterion (frequency domain approach) extract the information about stability & relative stability.

(i) Time Response	Frequency Response
(1) It is an analytical technique	(1) It is a graphical technique
(2) For higher order means (whose order is > (greater than) 3) this method is not suitable	(2) higher order method can be easily on this method
(3) It is more accurate	(3) It is less accurate
(4) It is the system response is analysed for the basic input normally used in control system. These i/p are step, ramp, parabolic	(4) Thus response to sinusoidal i/p of frequency ω can be easily obtained.

Frequency Response

The steady state response of a system due to a sinusoid i/p is called as frequency response.

$$\xrightarrow{x(t)} \boxed{g(t)} \xrightarrow{y(t)}$$

Frequency Response

$x(t)$ sinusoidal i/p function $y(t)$ o/p funct^n $g(t)$ transfer fn

$$G(s) = \frac{N(s)}{D(s)} = \frac{L\{y(t)\}}{L\{x(t)\}} = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+\alpha)(s+\beta)(s+\gamma)}$$

$$= \frac{Y(s)}{X(s)}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{N(s)}{(s+\alpha)(s+\beta)(s+\gamma)}$$

$$\Rightarrow x(t) = A\sin\omega t \Rightarrow X(s) = \frac{A\omega}{s^2+\omega^2}$$

$$Y(s) = \left[\frac{N(s)}{(s+\alpha)(s+\beta)(s+\gamma)\cdots}\right]\left(\frac{A\omega}{s^2+\omega^2}\right)$$

$$\Rightarrow Y(s) = \frac{A_1}{s+\alpha} + \frac{A_2}{s+\beta} + \frac{A_3}{s+\gamma} \cdots + \frac{B_1}{s+j\omega} + \frac{B_2}{s-j\omega}$$

$$\Rightarrow y(t) = A_1 e^{-\alpha t} + A_2 e^{-\beta t} + A_3 e^{-\gamma t} \cdots + B_1 e^{-j\omega t} + B_2 e^{j\omega t}$$

$$\Rightarrow \boxed{y_{ss}(t) = B_1 e^{-j\omega t} + B_2 e^{j\omega t}}$$

$$Y(s) = G(s)X(s) = G(s)\cdot\frac{A\omega}{s^2+\omega^2} = \frac{B_1}{s+j\omega} + \frac{B_2}{s-j\omega}$$

$$B_1 = G(s)\cdot\frac{A\omega}{s-j\omega}\Big|_{s=-j\omega} = \frac{-A}{j2}\,G(-j\omega)$$

$$B_2 = G(s)\frac{A\omega}{s+j\omega}\Big|_{s=+j\omega} = \frac{A}{j2}\,G(j\omega)$$

$$G(j\omega) = |G(j\omega)|e^{j\phi}$$

$$y_{ss}(t) = -\frac{A}{2j}|G(j\omega)|\,e^{-j\phi}\,e^{-j\omega t} + \frac{A}{2j}|G(j\omega)|\,e^{j\phi}e^{j\omega t}$$

$$= A|G(j\omega)|\left[\frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{j2}\right]$$

$$= A\,|G(j\omega)|\,\sin(\omega t+\phi)$$

$y(t) = A_1 e^{-\alpha t} \ldots + B_2 e^{j\omega t}$

∴ The steady state response of this eqn

$$y_{ss}(t) = B_1 e^{-j\omega t} + B_2 e^{j\omega t} = A\,|G(j\omega)|\,\sin(\omega t + \phi)$$

$$= |Y(j\omega)| \angle \phi$$

where $|Y(j\omega)| = |X(j\omega)|\,|G(j\omega)|$

$$\angle Y(j\omega) = \angle G(j\omega) + \angle X(j\omega) = \phi + 0 = \phi$$

$$|G(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|}$$

$$\angle G(j\omega) = \angle Y(j\omega) - \angle X(j\omega)$$

The frequency of sinusoidal o/p is the same as the i/p and there is only change in the amplitude & phase of the o/p with respect to i/p.

Methods in frequency domain

i) polar plot :- It is a plot between magnitude & phase angle where angular frequency (ω) varied from 0 to ∞.

ii) Bode plot :- It is plot of both ⓐ magnitude ⓑ phase veresus frequency ω in logarith scale.

iii) Magnitude versus phase plot :- It is a plot betn magnitude & phase on rectangular co-ordinates with frequency ω as variable

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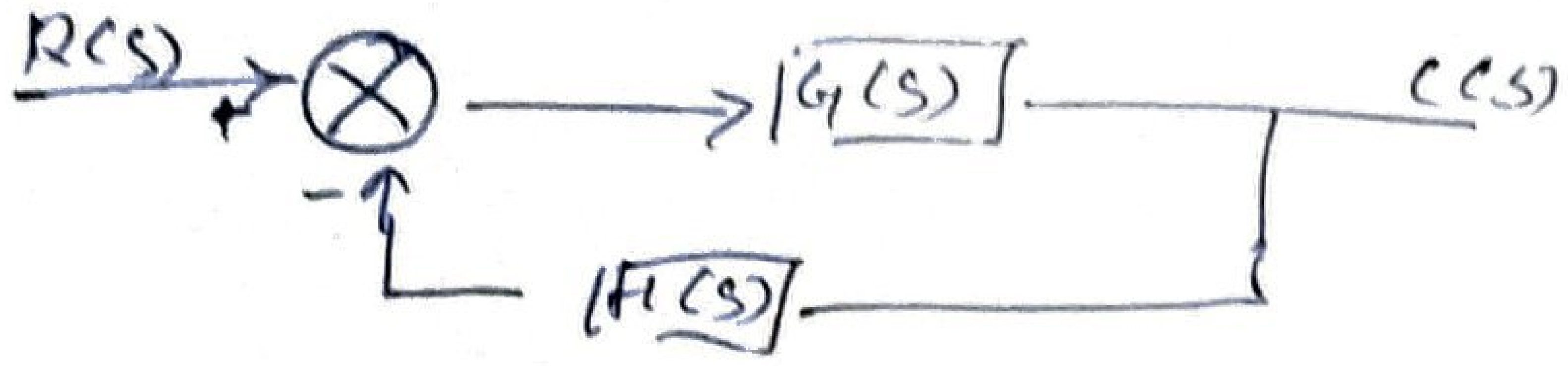
Frequency Response specification

① Resonance peak (m_r) :-



$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = m(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$\Rightarrow M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

The max value of $M(j\omega)$ as ω is varied is known as resonance peak M_r. The relative stability of closed loop system can be indicated by M_r. Generally M_r is betn 1.1 & 1.5

(ii) Resonance frequency :- The resonance at which resonance occure is known as resonance frequency.

(iii) Bandwidth :- ~~The slope of the log magnitude curve near the~~ The range of frequencies over which M is equal to greater than $\frac{1}{\sqrt{2}}$ is defined as Bandwidth (ω_b).

(iv) Cut-off Rate :- The slope of log magnitude curve near the cut-off frequency is known as cut-off rate. Cut-off rate can determined the ability of the system to distinguish a signal from noise.

(v) Phase margin :- phase margin is given by $180+\phi$, if ϕ is the phase angle of a system at unity gain. The frequency (ω_1) at which gain is unity is known as gain cross over frequency.

(vi) Gain Margin :- The gain margin is given by

$\frac{1}{|G(j\omega_1)H(j\omega_1)|}$ where ω_1 = phase cross over frequency.

The frequency (ω_1) at which the phase angle of the transfer function of the system is $(-180°)$ is known as phase cross over frequency. The increment required in gain to cause the system to become unstable is ... Gain Margin

CORRELATION BETWEEN TIME AND FREQUENCY DOMAIN SPECIFICATION FOR A SECOND ORDER SYSTEM:

For a second order system

$$G(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$H(s) = 1$

In frequency domain $s = j\omega$

$$G(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2} = \frac{\omega_n^2}{-\omega^2+2j\xi\omega_n\omega+\omega_n^2}$$

$$= \frac{1}{1+j2\xi\left(\frac{\omega}{\omega_n}\right)-\left(\frac{\omega}{\omega_n}\right)^2}$$

Let $u = \omega/\omega_n$

$$G(j\omega) = \frac{1}{1+2j\xi u-u^2}$$

$$G(j\omega) = |G(j\omega)| \angle G(j\omega) = M\angle\phi$$

where $|G(j\omega)| = M = \frac{1}{(1-u^2)^2+(2\xi u)^2} = \frac{1}{(1-u^2)^2+(2\xi u)^2}$

$$\angle G(j\omega) = \phi = -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

$$C(t) = \frac{1}{(1-u^2)^2+(2\xi u)^2} \sin\left[\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right]$$

= steady state o/p.

At peak value, the slope $\left.\frac{dM}{du}\right|_{\omega_r} = 0$

So the resonant frequency ω_r can be obtained as

$$\frac{dM}{du} = -\frac{1}{2}\frac{\left[2(1-u^2)(-2u) + 8\xi^2 u\right]}{\left[(1-u^2)^2 + (2\xi u)^2\right]^{3/2}}$$

At resonant frequency (ω_r) $\frac{dM}{du} = 0$

Put $u = u_r$

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{\left[2(1-u^2)(-2u) + 8\xi^2 u\right]}{\left[(1-u^2)^2 + (2\xi u)^2\right]^{3/2}}$$

$\Rightarrow 4u_r - 8u_r\xi^2 - 4u_r^3 = 0$

$\Rightarrow 4u_r(1-2\xi^2 - u_r^2) = 0$

$\Rightarrow u_r = 0$ & $1-2\xi^2 - u_r^2 = 0$ $\Rightarrow$ ~~$1-2\xi^2 = 0 \Rightarrow \xi^2 = 1/2$~~

$u_r = 0 \Rightarrow \frac{\omega_r}{\omega_n} = 0 \Rightarrow \omega_r = 0$

$1-2\xi^2 - u_r^2 = 0 \Rightarrow u_r^2 = 1-2\xi^2 \Rightarrow u_r = \sqrt{1-(2\xi)^2}$

$\Rightarrow \frac{\omega_r}{\omega_n} = \sqrt{1-(2\xi)^2}$

$\Rightarrow \omega_r = \omega_n\sqrt{1-(2\xi)^2}$

$\Rightarrow \omega_r = \omega_n\sqrt{1-(2\xi)^2}$

From eqn $M_r = G(j\omega)\big|_{u=u_r} = \frac{1}{\sqrt{(1-u_r^2)^2 + (2\xi u_r)^2}}$

$$\Rightarrow M_r = \frac{1}{\sqrt{(2\xi^2)^2 + 4\xi^2(1-2\xi^2)}}$$

$$= \frac{1}{\sqrt{4\xi^4 + 4\xi^2 - 8\xi^4}}$$

$$= \frac{1}{\sqrt{4\xi^2 - 4\xi^4}}$$

$$= \frac{1}{\sqrt{4\xi^2(1-\xi^2)}}$$

$$\Rightarrow M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

So M_r is a function of ξ

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From the eqn we have

$$\phi_r = \angle G(j\omega)\Big|_{u=u_r} = -\tan^{-1}\left(\frac{2\xi u_r}{(1-u_r^2)}\right)$$

$$\phi_r = -\tan^{-1}\frac{2\xi\sqrt{1-2\xi^2}}{2\xi^2}$$

$$\phi_r = -\tan^{-1}\frac{\sqrt{1-2\xi^2}}{\xi}$$

Conclusion

i) $M_r \to \infty$, $\omega_r \to \omega_n$ when $\xi \to 0$

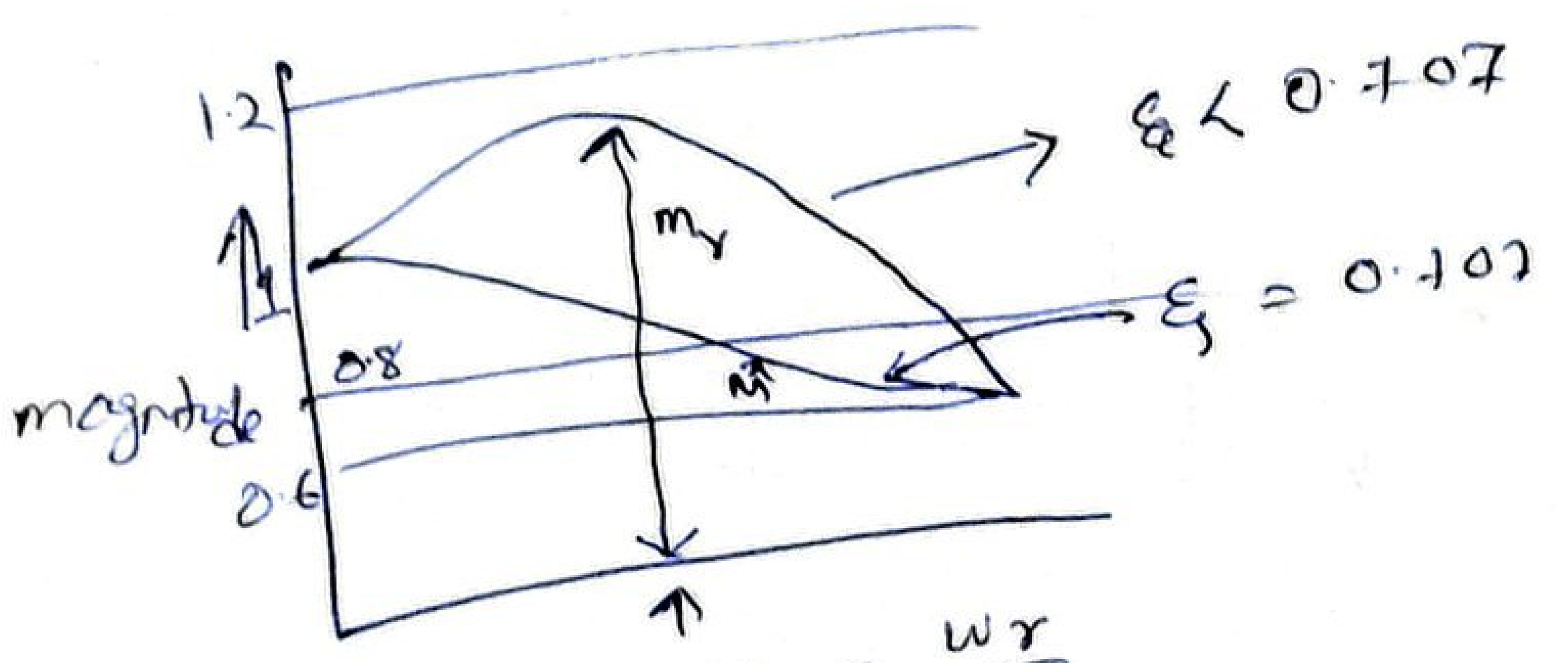
(ii) $M_r > 1$ $\omega_r < \omega_n$ for $0 < \xi < 0.707$

(iii) For $\xi > 0.707$

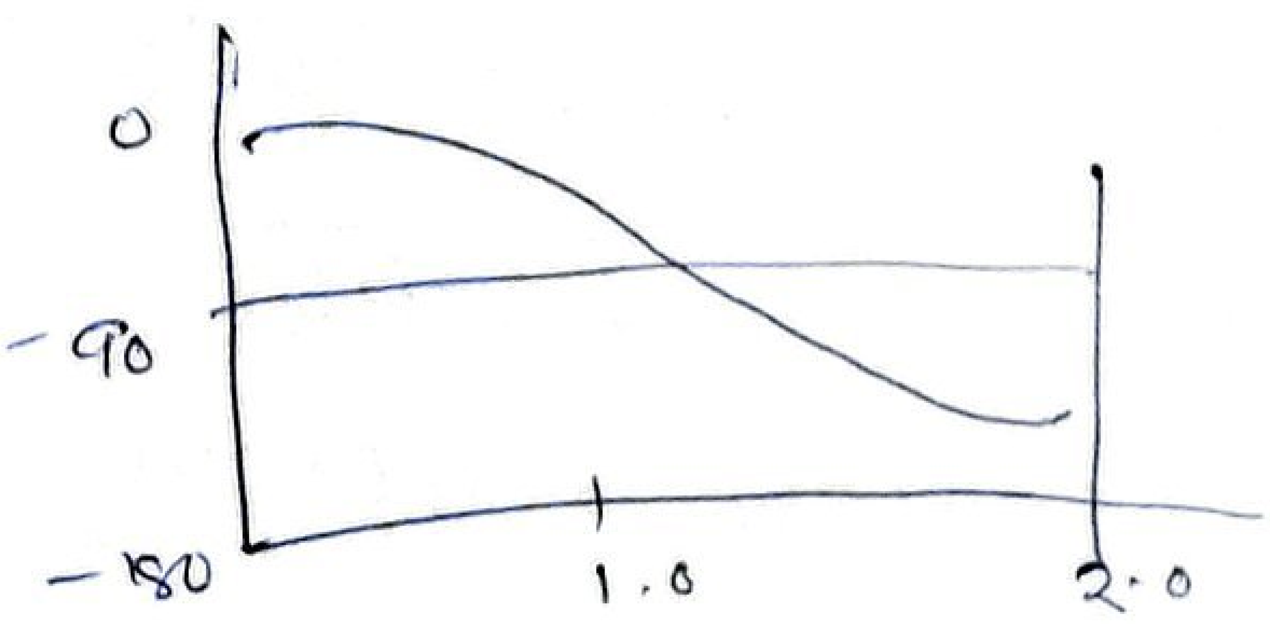
(a) There is no resonant peak.

(b) ω_r can't be zero

(c) There is no resonant peak such as the greatest value of $M = 1$.



magnitude characteristic in frequency domain



phase angle characteristic in frequency domain

For a second order system, the resonant peak M_r of its frequency response is indicative of its damping factor ξ for $0 < \xi < 0.707$. The resonant frequency ω_r of the frequency response is indicative of its natural frequency for a given ξ & is indicative of speed response ($t_s = \frac{4}{\xi\omega_n}$) hence indicative of the frequency response could thus be used M_r & ω_r of the frequency response could thus be used as performance indices for a 2nd order system.

For $\omega > \omega_r$, M decreases monotonically. The frequency at which M has a value of $\frac{1}{\sqrt{2}}$ of special significance & is called cut-off frequency ω_c. The signal frequencies above cut-off are attenuated in passing through a system.

→ For feed back control system, the range of frequencies over which M is equal to or greater than $\frac{1}{\sqrt{2}}$ is defined as bandwidth ω_b.

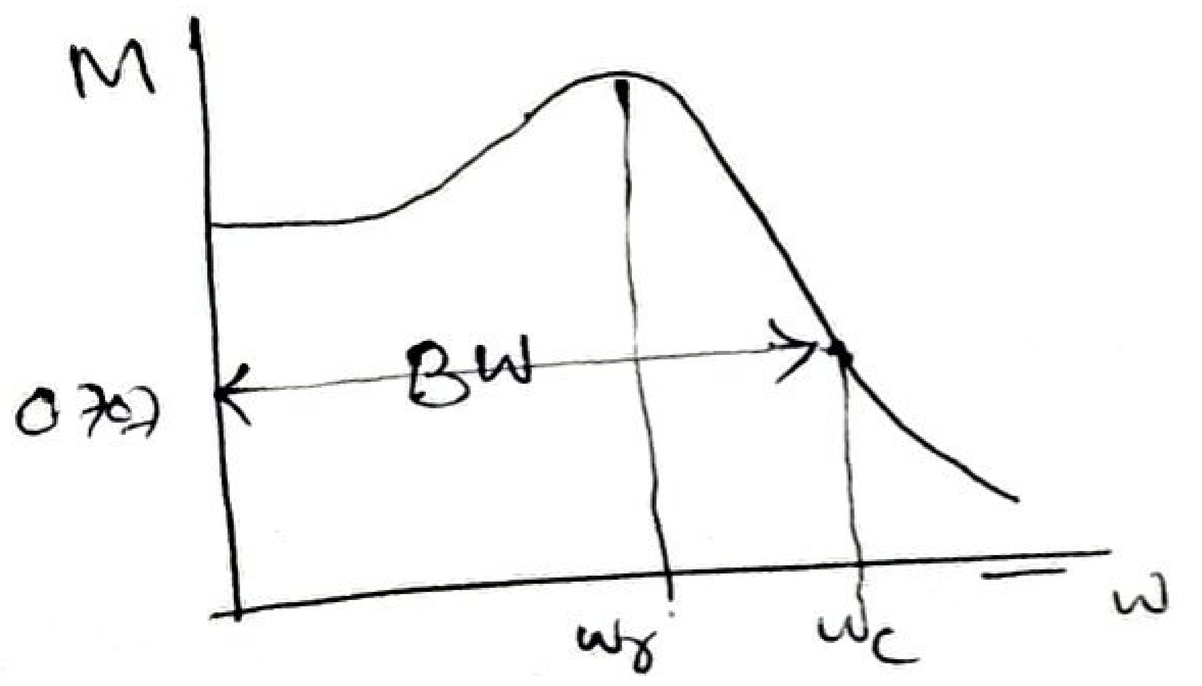
→ If a control system is a low pass filter (at zero frequency M = 1)

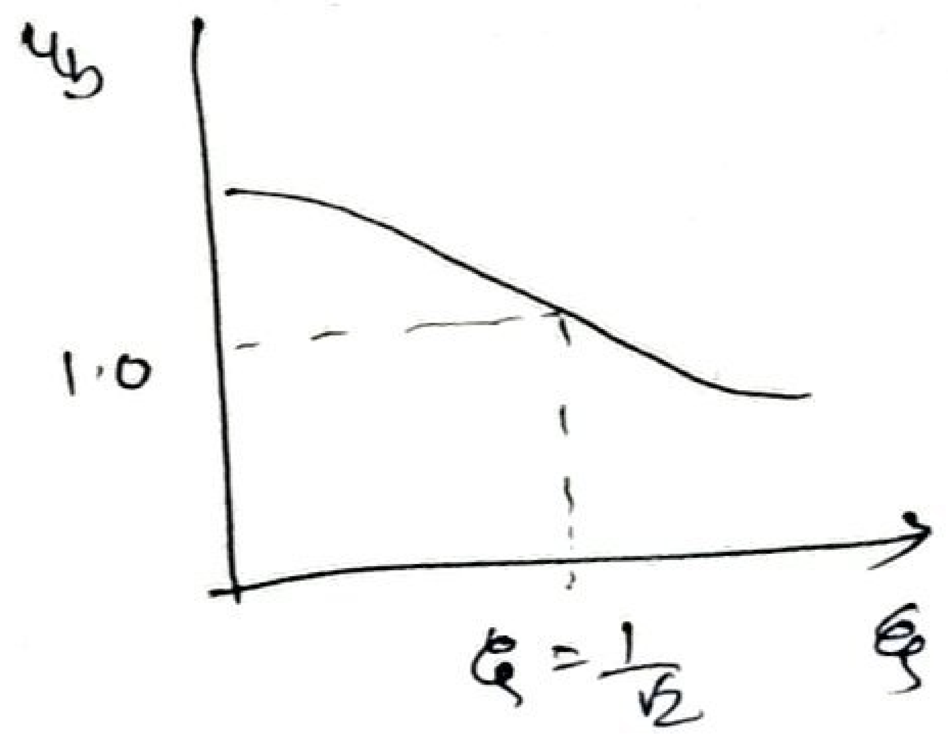
ω_b = bandwidth = ω_c = cut off frequency.

→ normalized bandwidth $u_b = \frac{\omega_b}{\omega_n}$

$$\Rightarrow M = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\xi u_b)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow u_b = \left[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\right]^{1/2}$$





BW versus damping factor.

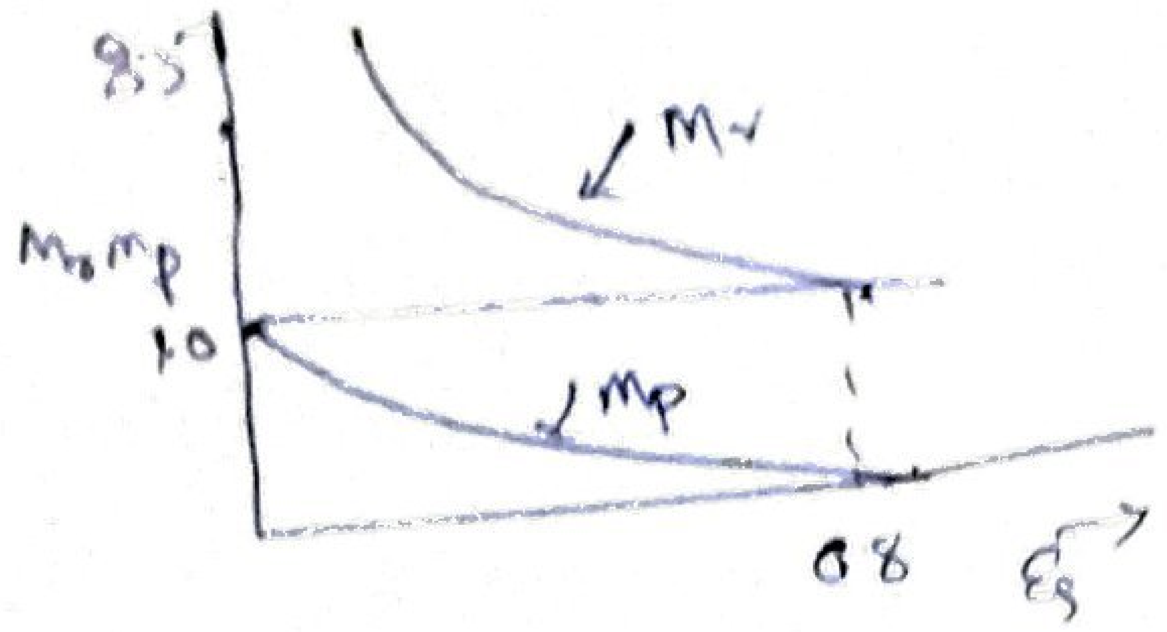
The denormalized bandwidth

$$\omega_b = \omega_n \quad 1 \quad 2 \quad ^2 \sqrt{2 \quad 4^2 \quad 4^4} \quad ^{1/2}$$

The expression for the damped frequency of oscillation ω_d & peak overshoot M_p of the step response for $0 < \xi < 1$ are

$$\omega_d = \omega_n\sqrt{1 \quad 2}$$

$$M_p = e^{\quad / \sqrt{1 \quad 2}}$$



Correlation betn M_r & M_p

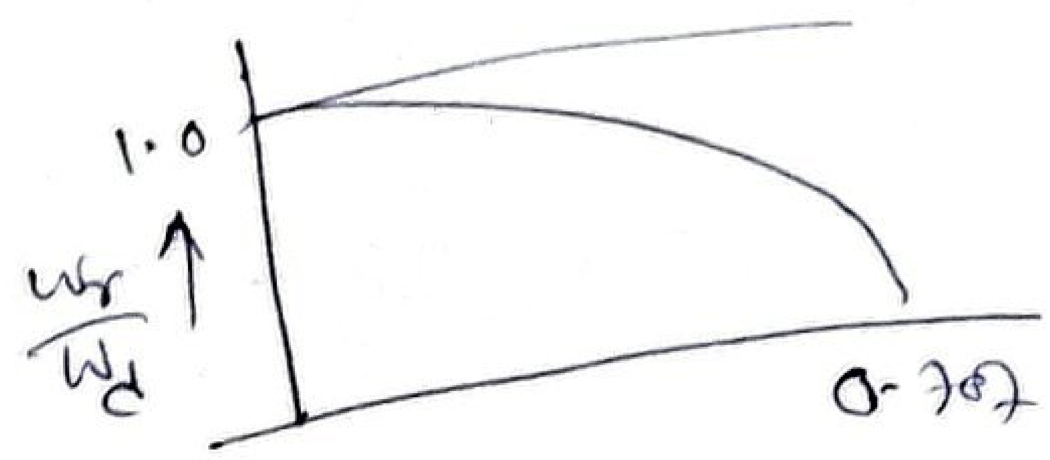
The comparison of M_r & M_p plots shows that for $0 < \xi < \frac{1}{\sqrt{2}}$ the two performance indices are co-related as both are function of the system damping factor ξ only.

It means that a system with a given value of M_r & its frequency response, must show a corresponding value of M_p, if subjected to a step input.

For $\xi > 1/\sqrt{2}$, the resonant peak M_r does not exist & correlation breaks down.

Co-rrelation betn ω_r & ω_d

$$\frac{\omega_r}{\omega_d} = \sqrt{\frac{1 - 2\xi^2}{1 - \xi^2}}$$



Correlation betn ω_b & ω_d

The bandwidth (ω_b) a frequency domain concept is indicative of undamped natural frequency of a system for a given ξ & therefore indicative of speed of response $\left(t_s = \frac{4}{\xi \omega_n}\right)$ a time domain concept.

Addition of Zero

If a zero is added in the forward path a 2nd order system, the following effect occur

(i) Rise time decreases due to increase in bandwidth

(ii) settling time increases

(iii) system will more stable.

Addition of pole

If a pole is added in forward path of 2nd order system

1. Rise time will increase & bandwidth decrease
2. M_r is increase.
3. System is less stable.

Advantage of frequency domain Analysis

1. It is possible to obtain a frequency response test with good accuracy in laboratory.
2. It is very useful when it is very difficult to obtain transfer function by an analytical method.
3. As compared to time domain, it is very easy to design open loop t/f for specified closed-loop performance in frequency domain.
4. It is very easy to visualize there the effect of noise disturbance & parameter variations in frequency domain

Disadvantage of frequency domain

1. It can be obtain for only linear system
2. Frequency response can be obtained only if the time constants are upto few minutes, otherwise it will be very time-consuming process.

Prob →

Determine the frequency domain specification for a 2nd order system with unity feedback &

$G(s) = 100/s(s+3)$

Ans $M(s) = \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{100/s(s+3)}{1+100/s(s+3)} = \frac{100}{s^2+3s+100}$

Comparing $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\xi\omega_n = 3 \Rightarrow \xi = 0.15$

$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 3.37$

$\omega_r = \omega_n\sqrt{1-2\xi^2} = 9.77$

$\omega_b = \omega_n\left[(1-2\xi^2)+\sqrt{4\xi^2-4\xi^2+2}\right]^{1/2} = 15.29$ rad/sec